(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2021 ONLY)

21PMS416

REG.NO. :

N.G.M. COLLEGE (AUTONOMOUS): POLLACHI

END-OF-SEMESTER EXAMINATIONS: MAY-2023

COURSE NAME: M.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: IV **TIME: 3 HOURS**

OPERATOR THEORY

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. (K1)

1. Elements of $NBV[a\,,b]$ are called ______ functions of bounded variation on [a,b].
   a) normalized b) semi type c)reflexive d) symmetric
2. Every Hilbert space is a _______________.
   a) transitive b) onto c) reflexive d)symmetric
3. Every bounded operator of finite rank is a _____________ operator.
   a) Compact b) compound c) normalized d) inclusion
4. Eigen spectrum is also known as _____ spectrum.
   a) point b) linear c) normalized d) nonlinear
5. Let X be a Hilbert space and $A \in B(X)$.Then A is said to be a self adjoint operator if________.
   a) $A = B$ b) $A^* = A$ c) $A' = A$ d) AXA

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Define function of bounded variation.
7. Define Reflexivity.
8. Define Banach algebra.
9. Define Eigen space.
10. Define unitary operator.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If $u_a = 0$, and for $a < t \le b$, $u_t = \chi_{(a,t]}$.Prove that for every f in the dual of $L^p[a,b], 1 \le p < \infty$, the function $v : [a,b] \uparrow\rightarrow \mathbb{K}$ defined by $v(t) = f(u_t), t \in [a,b]$, is isabsolutely continuous.

(OR)

b) Prove that a normal linear space is separable if its dual is separable.

12.a) If X is a reflexive , show that $X$is separable if and only if its dual X' is separable.

(OR)

b) Prove that every bounded sequence in a reflexive space has a weakly convergent subsequence.

(CONTD........2)

13.a) Prove that $K(X,Y)$ is a subspace of $B(X,Y)$.

(OR)

b) If X and Y be normed linear spaces and $A: X \to Y$ be an injective compact operator ,prove that $A^{-1}: R(A) \to X$ is continuous if and only if rank $A < \infty$.

14.a) Let X be a Banach space over $\mathbb{C}$ and $A \epsilon B(X)$. Prove that $\sigma(A) \neq \emptyset$.

(OR)

b) Suppose X is a Banach space, $A \in B(X)$ and $\lambda \epsilon \mathbb{K}$. Show that $\lambda \epsilon \sigma(A)$ if and only if either $\lambda \epsilon \sigma_{app}(A)$ or $R(A - \lambda I)$ is not dense in X.

15.a) Let X, Y be inner product spaces, and $A: X \to Y$ be a linear operator such that the adjoint A* exists. If $A \in B(X,Y)$, show that (i) $A^* \in B(X,Y)$, (ii) $\|A^*\| = \|A\|$ (iii) $\|A^*A\| = \|A\|^2$

(OR)

b) let X be a Hilbert space and $A \in B(X)$ be a self-adjoint operator. Show that $\|A\| = sup\{|\langle Ax, x\rangle| : x \in X, \|x\| = 1\}$.In particular,$A = 0$ if and only if$\langle Ax, x\rangle = 0$ for all $x \in X$.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS

(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. State and prove Fundamental theorem of Lebesgue integration. **K4**

17. Prove that the map $T: (l^p(n))' \to l^q(n)$ defined by $T(f) = (f(e_1), \dots \dots f(e_n)), f \in (l^p)'$, is a surjective linear isometry where $e_j \in \mathbb{K}^n$ is such that $e_j(i) = \delta_{ij}$ for $i, j = 1, \dots \dots n$. **K5**

18. Prove the following:
 i) Every closed subspace of a reflexive space is reflexive.
 ii) Every normed linear space which is linearly isometric with a reflexive space is reflexive.
 iii)A Banach space is reflexive if and only if its dual is reflexive. **K5**

19. Let X and Y be normed linear spaces. Prove the following **K5**
 i) If $A \in \mathcal{K}(Y, X)$, then $A' \in \mathcal{K}(Y', X')$
 ii) Converse of (i) holds if Y is a Banach space.

20. State and prove Spectral Mapping Theorem. **K4**

21. Let X be a Hilbert space and $A \epsilon B(X)$. Show that **K5**
 i) A is normal if and only if $\|Ax\| = \|A * x\|$ for every $x \in X$.
 ii) A is unitary if and only if A is surjective and $\|Ax\| = \|x\|$ for every $x \in X$.
 In particular, if A is a unitary operator, then $\|A\| = 1$.

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