(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2021 ONLY)

21UMS204

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : JULY-2022

B.Sc.-MATHEMATICS   MAXIMUM MARKS: 70
SEMESTER II   TIME : 3 HOURS

PART - III
ANALYTICAL GEOMETRY

SECTION - A   (10 X 1 = 10 MARKS)
ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS.   (K1)

1. The locus of the point of intersection of tangents at the extremities if chords through that point is taken as _____.
    1) Pole   2) Polar
    3) Polar of a point with regard to a Circle   4) Polar of an arbitrary point

2. What is the equation of the tangent to the circle $r = 2a \cos\theta$ at α ?
    1) $r\cos(\theta + 2\alpha) = 2a\cos^2\alpha$ 2) $r\cos(\theta - 2\alpha) = 2a\cos^2\alpha$
    3) $r\sin(\theta + 2\alpha) = 2a\sin^2\alpha$ 4) $r\sin(\theta - 2\alpha) = 2a\sin^2\alpha$

3. Which of the following statements represents one of the characteristics of the sphere?
    1) The coefficients of x^2, y^2, z^2 are equal in the equations of the sphere
    2) The terms xy, yz, zx are present
    3) The equation of the sphere of degree is x, y, z
    4) The equation of the sphere is a quadratic equation

4. How will be the section of a cone by any plane perpendicular to its axis?
    1) sphere   2) cylinder   3) cone   4) circle

5. The axis of the cylinder is parallel to _____ of the cylinder.
    1) the generator   2) circles   3) vertex   4) guiding curve

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES.   (K2)

6.   What is the condition for the lines $lx + my + n = 0$ and $l_1x + m_1y + n_1 = 0$ to be conjugate?
7.   Write the condition for the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ to be a tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$.
8.   Represent the equation of a circle on a sphere.
9.   Define Right circular cone.
10. Define ellipsoid.

SECTION – B   (5 X 4 = 20 MARKS)
ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Show that the conjugate lines through a focus of an ellipse are at right angles.
(OR)
    b) Show that the product of the perpendicular drawn from any point on a hyperbola to its asymptotes is a constant.

12.a)   Trace the curve $\frac{10}{r} = 3\cos\theta + 4\sin\theta + 5$.
(OR)
    b)   Find the locus of the foot of the perpendicular drawn from the pole to the tangents to the circle $r = 2a\cos\theta$.

13.a) Find the equation to the sphere through the four points $(2,3,1), (5,-1,2), (4,3,-1)$ and $(2,5,3)$

(OR)

b) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$ $2x - y + 2z = 5$ for a great circle.

14.a) Find the equations of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\dfrac{x}{32} = \dfrac{y}{72} = \dfrac{z}{72}$.

(OR)

b) Find the angle between the lines given by $x + y + z = 0, \dfrac{yz}{b-c} + \dfrac{zx}{c-a} + \dfrac{xy}{a-b} = 0$.

15.a) Find the equation of a right circular cylinder of radius 3 with axis $\dfrac{x+2}{3} = \dfrac{y-4}{6} = \dfrac{z-1}{2}$.

(OR)

b) Find the condition that the line $\dfrac{x-\alpha}{l} = \dfrac{y-\beta}{m} = \dfrac{z-\gamma}{n}$ should intersect the polar of the line $\dfrac{x-\alpha_1}{l_1} = \dfrac{y-\beta_1}{m_1} = \dfrac{z-\gamma_1}{n_1}$ with respect to the conicoid $ax^2 + by^2 + cz^2 = 1$.

SECTION - C   **(4 X 10 = 40 MARKS)**

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16ᵗʰ QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS
(FROM Qn. No : 17 to 21)   **(K4 (Or) K5)**

16. If OD is the diameter parallel to secant APQ through A meeting the conicoid at P and Q show that $\dfrac{AP.AQ}{OD^2}$ is constant.

17. Find the locus of the middle point of chords of the circle $x^2 + y^2 = a^2$ which subtend a right angle at the point (h, k).

18. If the normal at α, β, γ on $\dfrac{l}{r} = 1 + \cos\theta$ meet in the point (p, φ) then show that

(i) $\tan\dfrac{\alpha}{2} + \tan\dfrac{\beta}{2} + \tan\dfrac{\gamma}{2} = 0$ (ii) $\alpha + \beta + \gamma = 2n\pi + 2\varphi$

19. Find the condition that the line $\dfrac{x-a}{l} = \dfrac{y-b}{m} = \dfrac{z-c}{n}$ where $l^2 + m^2 + n^2 = 1$ should touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. Show that there are two spheres through the points (0, 0, 0), (2a, 0, 0), (0, 2b, 0) which touch the above line and that the distance between their centres is $\dfrac{2}{n^2}[c^2 - (a^2 + b^2 + c^2)n^2]^{1/2}$

20. Find the condition for the equation F(x, y, z) = $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy$ $+2ux + 2vy + 2wz + d = 0$ to represent a cone. If it represents a cone, show that the co-ordinates of the vertex satisfy the equations $\dfrac{\partial F}{\partial x} = \dfrac{\partial F}{\partial y} = \dfrac{\partial F}{\partial z} = \dfrac{\partial F}{\partial t} = 0$ where t is used to make F(x, y, z) homogenous and is equated to unity after differentiation.

21. Find the locus of the point of intersection of three mutually perpendicular tangent planes to the central conicoid $ax^2 + by^2 + cz^2 = 1$.

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