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(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2022 ONLY)

22UMS203

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY-2023

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 50**

SEMESTER: II **TIME : 3 HOURS**

PART - III

TRIGONOMETRY, VECTOR CALCULUS AND FOURIER SERIES

SECTION – A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.

MULTIPLE CHOICE QUESTIONS. (K1)

1. $\sin\theta =$ _____

   a) $\frac{e^{i\theta}-e^{-i\theta}}{2i}$ b) $\frac{e^{i\theta}+e^{-i\theta}}{2i}$ c) $\frac{e^{i\theta}-e^{-i\theta}}{2}$ d) $\frac{e^{i\theta}-e^{-i\theta}}{2i^2}$

2. A sequence in which $a_{n+1} \geq a_n$ for all values of n is called

   a) Cauchy's sequence b) Monotonic decreasing sequence

   c) Monotonic increasing sequence d) None of the above

3. $\int_0^\pi \cos mx\, dx =$ _____ if m is an integer.

   a) 0 b) ∞ c) 1 d) 2

4. $\nabla f(r) \times \vec{r} =$ _______.

   a)1 b) 2 c) 0 d) -1

5. $\nabla^2\phi = 0$ is called ______.

   a) logarithmic b) exponential c) hyperbolic d) harmonic

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Find the Value of $x^n + \frac{1}{x^n}$ if $x = cos\theta + isin\,\theta$

7. Write an example for oscillate sequence.

8. Define odd function.

9. Define Solenoidal vector.

10. State Green's theorem.

SECTION – B (5 X 3 = 15 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Express $cos\ 8\theta$ in terms of $\sin\theta$

(OR)

b) If $\cosh u = \sec\theta$, show that $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

(CONTD…..2)

12. a)If $u_1 + u_2 + \cdots + u_n + \cdots$ is convergent, then $\lim_{n\to\infty} u_n = 0$.

(OR)

b) Show that the series $\sum \frac{\{(n+1)r\}^n}{n^{n+1}}$ is convergent if $r < 1$ and divergent if $r \geq 1$.

13. a)Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series in the interval $0 \leq x \leq 2\pi$

(OR)

b) Express $f(x) = x, -\pi < x < \pi$ as a Fourier series with period 2π.

14. a)Find ϕ if $\nabla\phi = (y + \sin z)\vec{i} + x\vec{j} + x\cos z\,\vec{k}$.

(OR)

b)Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla.\vec{F}) - \nabla^2\vec{F}$ where $\vec{F}$ is a vector point function.

15. a)If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_C \vec{F}.d\vec{r}$ where C is the curve on the xy plane $y = 2x^2$ from (0, 0) to (1, 2).

(OR)

b)For the vector function $\vec{F} = 2xy\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 + 1)\vec{k}$, determine the value of the integral $\int_C \vec{F}.d\vec{r}$ around the unit circle with centre at the origin in the xy plane.

SECTION – C　　(5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

(K4 (Or) K5)

16. a)Expand $\sin^3\theta\cos^5\theta$in series of sines of multiples of θ.

(OR)

b)If $\cosh(a + ib)\cos(c + id) = 1$, prove that

(1)$\cos b\cos c\cosh a\cosh d + \sin b\sin c\sinh a\sinh d = 1$

(2) $\tanh a\tan b = \tanh d\tan c$

17. a)Prove that the series $1 + \frac{1}{2}.\frac{a}{b} + \frac{1.3}{2.4}\frac{a(a+1)}{b\,(b+1)} + \frac{1.3.5}{2.4.6}\frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \cdots$ is convergent if $a > 0$, $b > 0$ and $b > a + \frac{1}{2}$

(OR)

b) Settle the range of values of x for which the following series converge:

(1)$\sum \frac{x^n}{1+x^n}$　　(2) $\sum \frac{x^n}{1+n^2x^{2n}}$

18. a)Find in the range $-\pi$ to π, Fourier series for $\begin{matrix} y = 1 + x & 0 < x < \pi \\ y = -1 + x & -\pi < x < 0 \end{matrix}$

(OR)

b)Show that $x^2 = \frac{\pi^2}{3} + 4\sum(-1)^n\frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$.

(CONTD…..3)

19. a)A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 5$ where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\vec{i} + \vec{j} + 3\vec{k}$.

(OR)

b)A field $\vec{F}$ is of the form $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$. Show that $\vec{F}$ is conservative field and find a function ϕ such that $\vec{F} = \nabla\phi$.

20. a)Verify Gauss's theorem for $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ taken over the region bounded by the planes

$$x = 0, x = a, y = 0, y = a, z = 0, z = a.$$

(OR)

b)If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, evaluate $\int \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 0, 0) along the following paths

a) $x = t, y = t^2, z = t^3$

b) the straight lines from (0, 0, 0) to (1, 0, 0) then to (1, 1, 0) and then to (1, 1, 1).

c) the straight line joining (0, 0, 0) to (1, 1, 1).

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ETHICAL PAPER