(NO. OF PAGES: 2)

(FOR THE CANDIDATES ADMITTED

DURING THE ACADEMIC YEAR 2020 ONLY)

20UMS615 / 20UMA615

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS :MAY - 2023

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: VI **TIME : 3 HOURS**

PART – III

COMPLEX ANALYSIS

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. **(K1)**

1. A function that satisfies Laplace Equation is called as___________________
   a. holomorphic b. harmonic c. conjugate d. Riemann

2. Every Cauchy sequence is ______________________________
   a. connected b. convergent c. divergent d. alternating

3. The smallest positive period of e^{iz} is denoted by ________________
   a. π b. 2π c. 3π d. $\pi/2$

4. If the closed curve γ lies inside a circle then the value of $n(\gamma,a)$ for all the points outside the circle is ______________
   a. 1 b. -1 c. ∞ d. 0

5. If $f(z)=z^2 \sin z$ then the zero of order for f(z) is ___________.
   a. 2 b. 3 c. 4 d. 0.

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. **(K2)**

6. Write the Cauchy-Riemann differential equation?
7. When is a sequence said to be uniformly convergent?
8. What is the expansion of $\log(z_1 z_2)$?
9. State the condition for the function f to be exact differentiable.
10. Given an example for a simple pole.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) If g(z) and f(z) are analytic functions show that g(f(z)) is also analytic.

(OR)

b) Show that if all zeros of a polynomial P(z) lie in a half plane, then all zeros of the derivative P'(z) lie in the same half plane.

(CONTD……..2)

12.a) Discuss the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ for real values of x

(OR)

b) Determine the radius of the convergence of the power series $\sum n^p\, z^n$

13.a) Find the values of tan (1+i)

(OR)

b) Express arc tan w in terms of the logarithm

14. a) If f (z) is analytic in an open disc Δ then $\int f(z)dz = 0$ for every curve γ in Δ.

(OR)

b) Compute the value of $\int \frac{dz}{Z^2-1}$ for the positive sense of the circle $|z| = 2$.

15. a) Show that a non-constant analytic function maps open sets to open sets

(OR)

b) Show that if f(z) is analytic and non-constant in a region Ω then its absolute value $|f(z)|$ has no maximum value in Ω

SECTION - C (4X10= 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16thQUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS
(FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. Check whether the following functions are differentiable
    (i) $f(z) = e^x(\cos y - i\sin y)$
    (ii) $f(z) = z^3$

17. Verify Cauchy –Riemann's equation for the functions z^2 and z^3.

18. Prove the Abel's limit theorem with suitable arguments.

19. Show that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant.

20. Substantiate that if the function f(z) is analytic then the value of the integral f(z) over ∂R is zero.

21. State and prove the Taylor's theorem.

****

ETHICAL PAPER