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(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2020 ONLY)

20UMS613 / 20UMA613

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY - 2023

COURSE NAME: B.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**

SEMESTER: VI **TIME : 3 HOURS**

PART - III
LINEAR ALGEBRA

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. An elementary matrix is ______________________
   a. an identity matrix b. row echelon matrix c.both a and b d. invertible

2. If V be a vector space over F, then any subset S of V containing the zero vector is _____________.
   a. linearly independent b. empty c. linearly dependent d.finite

3. If T is a linear transformation from V onto W then T is ________________
   a. invertible b. singular c. non-singular d. both a and b

4. If W_1 and $W_2$are the subspaces of a finite dimensional vector space V then __________
   a. $(W_1 + W_2)^0 = W_1^0 + W_2^0$ b. $(W_1 + W_2)^0 = W_1^0 \cup W_2^0$
   c. $(W_1 - W_2)^0 = W_1^0 - W_2^0$ d. $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$

5. If T^t is the transpose of the linear transformation of T then _______________
   a. $(T^t g)(\alpha) = g(T\alpha)$ b. $(T^t g)(\alpha) = g(T)$ c.$(T^t g)(\alpha) = g(\alpha T)$ d.$(T^t g)(\alpha) = \alpha(Tg)$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES (K2)

6. Give an example for an elementary matrix.
7. Define Symmetric matrix.
8. Is $T:R^2 \to R^2$ defined by $T(a,b) = (2a-3b , a+4b)$ a linear transformation**?**
9 .Define dual space of a vector space.
10. What is the nature of hyperspace of a vector space?

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Let A and B be n ×n matrices over F. Show that if A is invertible so is A^{-1} and $(A^{-1})^{-1} = A$.

(OR)

b) Show that if A is a n ×n matrix, then A is invertible and A is a product of elementary matrices.

(CONTD…..2)

12.a) Prove that if A is a m ×n matrix over F and B, C are n ×p matrices over F, then
$A(dB+C) = d(AB) + AC$ for each scalar d in F

(OR)

b) Show that the intersection of any collection of subspaces of a vector space V is a subspace of V

13. a) Prove that the inverse of the invertible linear transformation T from the vector space W onto V is also a linear transformation.

(OR)

b) Show that a linear transformation T from V into W is non-singular iff T carries each linearly independent subset of V onto a linearly independent subset of W

14. a) Let W be a subspace of R^5 which is spanned by the vectors
$\alpha_1 = (2,-2,3,4,-1)$, $\alpha_2= (-1,1,2,5,2)$, $\alpha_3= (0,0,-1,-2,3)$, $\alpha_4= (1,-1,2,3,0)$. Determine the annihilator of W.

(OR)

b) If f is a linear functional on R^3 and if $\alpha_1 = (1,0,1)$, $\alpha_2= (0,1,-2)$, $\alpha_3= (-1,-1,0)$, find $f(\alpha)$ where $\alpha= (a,b,c)$ and given $f(\alpha_1) = 1, f(\alpha_2) = -1$ and $f(\alpha_3) = 3$

15. a) Show that f in V* is a mapping $\alpha\rightarrow L_\alpha$ is an isomorphism of V onto V**, where V is a finite dimensional vector space over the filed F and each vector α in V is defined as
$L_\alpha(f) = f(\alpha)$

(OR)

b) Prove that each basis for V* is the dual of some basis for V, where V is a finite dimensional vector space over the field F.

SECTION - C (4X10= 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16^{th}QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS
(FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. Consider a system of equations
$x_1 - x_2 +2x_3 = 1$; $2x_1 +2x_3 =1$; $x_1-3x_2 +4x_3 = 2$. Does this system have a solution? If so, describe explicitly all solutions

17. Check whether the given matrix $\begin{bmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{bmatrix}$ is invertible

18. Prove that if W1 and W2 are finite dimensional subspaces of a vector space V then W1 + W2 is finite dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$

19. Show that every n-dimensional vector space over the filed F is isomorphic to the space F^n

20. Prove that $\dim W + \dim W^0 = \dim V$, where V is a finite dimensional vector space over the Field F and W is the subspace of V

21. Show that if S is any subset of a finite dimensional vector space V, then $(S^0)^0$ is the subspace spanned by S

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