(FOR THE CANDIDATES ADMITTED DURING THE ACADEMIC YEAR 2021 ONLY)

21UMS203

REG.NO. :

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI
END-OF-SEMESTER EXAMINATIONS : JULY- 2022
B.Sc.-MATHEMATICS **MAXIMUM MARKS: 70**
SEMESTER: II **TIME : 3 HOURS**

PART - III

TRIGONOMETRY, VECTOR CALCULUS AND FOURIER SERIES

SECTION - A (10 X 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS.
MULTIPLE CHOICE QUESTIONS. (K1)

1. What is the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$?
   1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{3\pi}{2}$ 4) π
2. What is known as Euler's formula?
   1) $e^{i\theta} = \cos\theta + i\sin\theta$ 2) $e^{i\theta} = \cos\theta - i\sin\theta$
   3) $e^{-i\theta} = \cos\theta + i\sin\theta$ 4) $e^{-i\theta} = -\cos\theta - i\sin\theta$
3. When it is true that $\int_{\lambda}^{\lambda+2\pi} \sin nx\, dx = 0$?
   1) n is a rational number 2) n is an irrational number
   3) n is an integer 4) n is a negative integer
4. How can the operator ∇ be applied on $\frac{\varphi}{\Psi}$?
   1) $\frac{\Psi(\nabla\varphi)-\varphi(\nabla\Psi)}{\Psi^2}$ 2) $\frac{\Psi(\nabla\Psi)-\varphi(\nabla\varphi)}{\Psi^2}$
   3) $\frac{\varphi(\nabla\varphi)-\Psi(\nabla\Psi)}{\Psi^2}$ 4) $\frac{\Psi(\nabla\varphi)+\varphi(\nabla\varphi)}{\Psi^2}$
5. What is the necessary and sufficient condition for $\int_{C(A_1,A_2)} \bar{f}.d\bar{r}$ to be independent of the path of integration?
   (1) $f = (\nabla\varphi)^2$ (2) $f = (\nabla\varphi)^3$ (3) $f = (\nabla\varphi)^4$ (4) $f = \nabla\varphi$

ANSWER THE FOLLOWING IN ONE (OR) TWO SENTENCES. (K2)

6. Expand $\cos n\theta$ in terms of trigonometric series.
7. Express $\cosh^2 x + \sinh^2 x$ in terms of suitable function.
8. What is the necessary and sufficient condition for a function to be an even function?
9. Define Harmonic function.
10. Define conservative field and scalar potential.

SECTION – B (5 X 4 = 20 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS. (K3)

11. a) Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$.

(OR)

b) Expand $\sin^3\theta\cos^5\theta$ in a series of sines of multiples of θ.

(CONTD….2)

12. a) If cos hu = sec θ then show that u = log tan $\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

(OR)

b) Express $\cosh^6\theta$ in terms of hyperbolic cosines of multiples of θ.

13. a) Find on expanding $f(x)=\frac{1}{2}(\pi-x)$ as Fourier series with period 2π.

(OR)

b) Find a Sine series for f(x) = c in the range 0 to π.

14. a) Find the directional derivative of $\varphi = 3xy^2 - x^2yz$ at the point (1, 2, 3) in the direction of $\vec{i}-2\vec{j}+2\vec{k}$.

(OR)

b) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$.

15. a) If $\bar{F}=z(x\vec{i}+y\vec{j}+z\vec{k})$ then evaluate $\int_C \bar{F}.d\bar{r}$ along the curve: x = t, y = t^2, z = t^3 from (0, 0, 0) to (1, 1, 1)

(OR)

b) Show that $\bar{f}=(y+y^2+z^2)\vec{i}+(x+z+2xy)\vec{j}+(y+2zx)\vec{k}$ is conservative and find its scalar potential.

SECTION - C (4 X 10 = 40 MARKS)

ANSWER ANY FOUR OUT OF SIX QUESTIONS
(16th QUESTION IS COMPULSORY AND ANSWER ANY THREE QUESTIONS (FROM Qn. No : 17 to 21) (K4 (Or) K5)

16. Find a cosine series in the range 0 to π for

$$f(x)=\begin{cases} x & 0<x<\frac{\pi}{2} \\ \pi-x & \frac{\pi}{2}<x<\pi \end{cases}.$$

17. Show that $\text{Cos}\frac{\pi}{9}.\text{Cos}\frac{2\pi}{9}.\text{Cos}\frac{4\pi}{9}=\frac{1}{8}$.

18. Separate into real and imaginary parts $\tan^{-1}(x+iy)$.

19. Find the Fourier series in the range $-\pi$ to π for $\begin{cases} y=1+x & 0<x<\pi \\ y=-1+x & -\pi<x<0 \end{cases}$

20. a) If $\nabla\varphi=(y+y^2+z^2)\vec{i}+(x+z+2xy)\vec{j}+(y+2zx)\vec{k}$ and if $\varphi(1,1,1)=3$ find φ.
b) Show that $(\nabla\varphi)$ x $(\nabla\varphi)$ is solenoidal.

21. Verify divergence theorem, for $\bar{A}=2x^2y\vec{i}-y^2\vec{j}+4z^2\vec{k}$ and for the region in the first quadrant bounded by the cylinder $y^2+z^2=9$ and the planes x = 0 and x = 2.

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